The Structural Complexity of Mammal´s Brain

A Formalistic Study

Rudolf Starkermann

•• FEEDCROSS ••

Production: Editions à la Carte Zürich

ISBN: 978-3-905708-58-5

Content

The Structural Complexity of Mammal´s Brain

Abstract

It was found that the structural formation of the brain of mammals, that is its neurological control organisation, is similar to the structure of technical, automatic control systems. A similar structure was found that applies to social, interrelated behaviour. In technical, continuously functioning, multivariable automatic control systems, an enormously large number of closed loops can be traced. In all these loops, the information is of constancy in change, which means that the information changes continuously and simultaneously in all parts of the system. - By induction, a formula is derived with which the total number of all loops in any size of a generally interwoven, multivariable control system can be determined. - The technical, automatic control system of one loop exemplifies the general concept of such kind of continuous functional behaviour. Assuming that one loop is defined as the *unit of structural complexity*, then the total number of loops of an entangled system reveals a basic measure of its intricacy. This number is particularly interesting, because the similarity of the structural formation of the brain of mammals with the structure of technical, industrial control systems, or social multiply goal-oriented organisations, can offer a measure of the trickiness of our psycho-social conglomerate. - However, the structural complexities of all three kinds, the technical, the social, and the cerebral one do not contain the dynamic characters, which are, indeed, of a much higher level of complexity than the structural ones.

1. A Comparison of Neurological Networks and Automatic Control Systems

The structural element of the central nervous system of mammals is represented with Figure 1. The nerve fibres, known as dendrites D, transmit stimuli, which are sent out by neurons. These signals are manipulated when passing through the synapse S. The neuron N collects the incoming signals and manipulates them again (this manipulation is still unknown). Only through one channel, called the axon A, information is leaving the neuron N. This axon branches into dendrites, leading to other neurons.

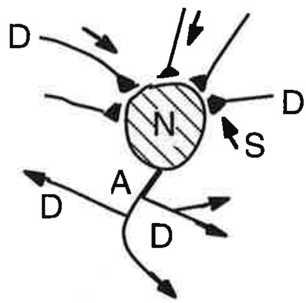

Figure 1: Structural element of the central nervous control system. (Ganglia are disregarded)

Figure 2 shows an ensemble of 4 elements of Figure 1. The heavy lines are forming a circular loop including two neurons, N_1 and N_2. We call now such a loop an *organisational unit*. The signal u, coming through an efferent fibre, can be a request for action; the signal x is considered to be the response upon u; and the signal d is considered to be a disturbance signal, which is not related directly to the neuron´s task. This supposed description is paralleled to the terms of a technical control loop.

The structure of a technical control loop is diagrammed with Figure 3. The task is to produce x(t), so that it approaches the requested u as close as possible, despite the disturbance signal d which tends to drive x(t) away from its way to u. C is the controller that leads the variable x(t) of the process P towards the fulfilment of the required goal u. The summing point $\Sigma 1$ measures continuously the difference between u and the actual x(t). This difference is called error $\varepsilon(t)$, the amount that is to become a minimum or even zero.

$$u - x(t) \rightarrow \varepsilon(t); \ \ \varepsilon(t) \rightarrow 0.$$

If the loop is capable to reduce the error ε(t) to zero, the controlled variable becomes equal to the request u. Such a one–loop system is the basic organisational unit in the realm of automatic controls. It illustrates in the simples form a two-neuron system. As an example, Figure 3 might represent a heat control of a room. The request u is the room temperature that has to be controlled; x(t) is the actual temperature in the room; the disturbance d can be an open window, letting cold air in the room; C is the controller that supplies the heating material; P is the air in the room that is to be heated.

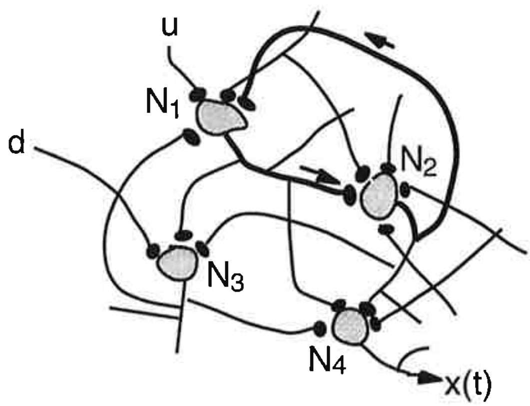

Figure 2: Network of 4 elements of Figure 1.

The degree of complexity of one organisational unit has been designated as 1. If a somewhat larger organization has two simultaneous goals, u_1 and u_2, it then also has two goal-striving variables, $x_1(t)$ and $x_2(t)$. Under these circumstances, the system can be disturbed by two (or more) signals, d_1 and d_2. The structure of such an organization is shown with Figure 4. Such a system is already complex and not simply an addition of two loops. There is mutual interaction between the two loops L_1 and L_2, thus rendering

10 more loops, lined up as Figure 5. The organisational complexity of Figure 4 is called 12, because it consists of 12 loops.

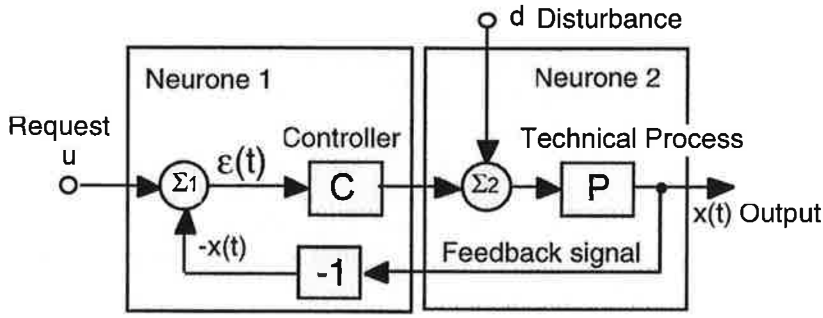

Figure 3: Principal schematic diagram of a one–loop automatic control system, the organisational unit.

Functions occurring in organizations of interacting loops are so complex that they can no longer be perceived visually. Functioning happens simultaneously and not in a step-by-step sequence of one step after the other as the digital computer and the human thinking concept operate. Therefore, only a mathematical investigation into the totality of such a system allows insight into its behaviour. Naturally, the laws that describe all the elements of a system and their interactions have to be known, otherwise, only a trial and error method is possible.

As an example of a technical plant with two variables to be controlled, Figure 6 is depicted. It shows the structure of a power plant of a paper mill together with its control system with its two variables. Two quantities of the plant have to be controlled automatically, whereas in a human body there are thousands of quantities that must be kept under constant and stable control. In biology, this control is called homeostasis.

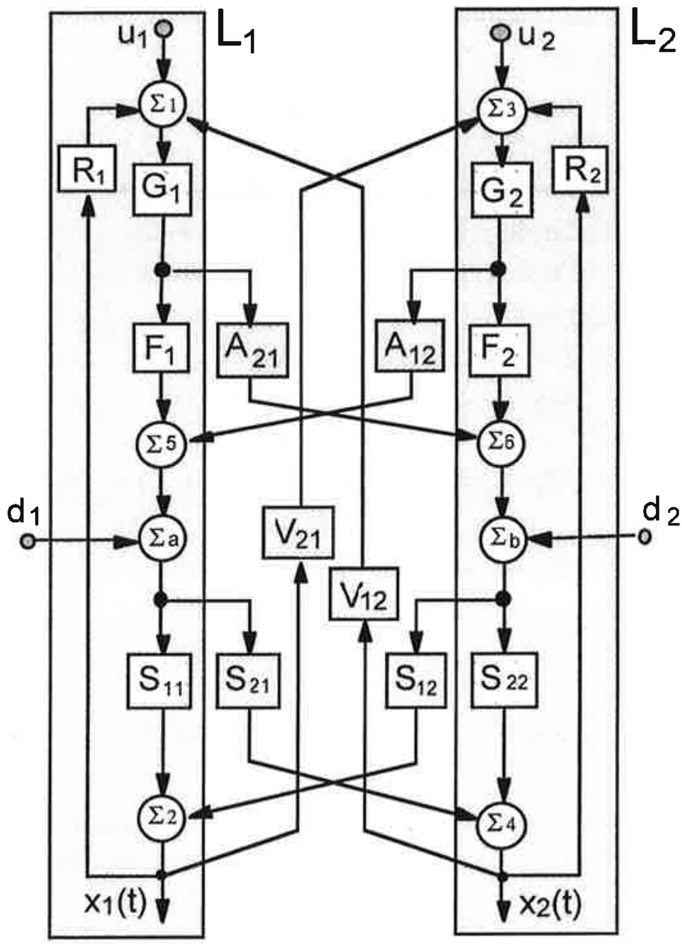

Figure 4: Principal diagram of a dual control system. The two loops, L_1 and L_2, have 3 bilateral interactions with each other.

1) $\Sigma1-G_1-F_1-\Sigma5-\Sigma a-S_{11}-\Sigma2-R_1-\Sigma1$

2) $S1-G_1-F_1-\Sigma5-\Sigma a-S_{21}-\Sigma4-V_{12}-\Sigma1$

3) $\Sigma1-G_1-A_{21}-\Sigma6-\Sigma b-S_{12}-\Sigma2-R_1-\Sigma1$

4) $\Sigma1-G_1-A_{21}-\Sigma6-\Sigma b-S_{22}-\Sigma4-V_{12}-\Sigma1$

5) $\Sigma a-S_{11}-\Sigma2-V_{21}-\Sigma3-G_2-A_{12}-\Sigma5-\Sigma a$

6) $\Sigma a-S_{21}-\Sigma4-R_2-\Sigma3-G_2-A_{12}-\Sigma5-\Sigma a$

7) $\Sigma b-S_{12}-\Sigma2-V_{21}-\Sigma3-G_2-F_2-\Sigma6-\Sigma b$

8) $\Sigma b-S_{22}-\Sigma4-R_2-\Sigma3-G_2-F_2-\Sigma6-\Sigma b$

9) $\Sigma1-G_1-F_1-\Sigma5-\Sigma a-S_{11}-\Sigma2-V_{21}-\Sigma3$
$-G_2-F_2-\Sigma6-\Sigma b-S_{22}-\Sigma4-V_{12}-\Sigma1$

10) $\Sigma1-G_1-F_1-\Sigma5-\Sigma a-S_{21}-\Sigma4-R_2-\Sigma3$
$-G_2-F_2-\Sigma6-\Sigma b-S_{12}-\Sigma2-R_1-\Sigma1$

11) $\Sigma1-G_1-A_{21}-\Sigma6-\Sigma b-S_{12}-\Sigma2-V_{21}-\Sigma3$
$-G_2-A_{12}-\Sigma5-\Sigma a-S_{21}-\Sigma4-V_{12}-\Sigma1$

12) $\Sigma1-G_1-A_{21}-\Sigma6-\Sigma b-S_{22}-\Sigma4-R_2-\Sigma3$
$-G_2-A_{12}-\Sigma5-\Sigma a-S_{11}-\Sigma2-R_1-\Sigma1$

Figure 5: The 12 loops of Figure 4.

The basic structure of the dual control system, Figure 4, with its 12 loops looks rather simple compared with the more detailed technical system Figure 6. In this Figure 6, it can be seen that lines that are conducting variables meet in elements, marked as Σ, and that each Σ-element has only one output. This output quantity is also a line-conductor. It leads its variable to other Σ-elements, forming this way an immense amount of closed information loops. (In Figure 6, the two controllers G_1 and G_2, the Valves V1 und V2, as well as the two measuring devices M_1 ND M_2 are not structured in their details; they are only depicted in concentrated blocks.)

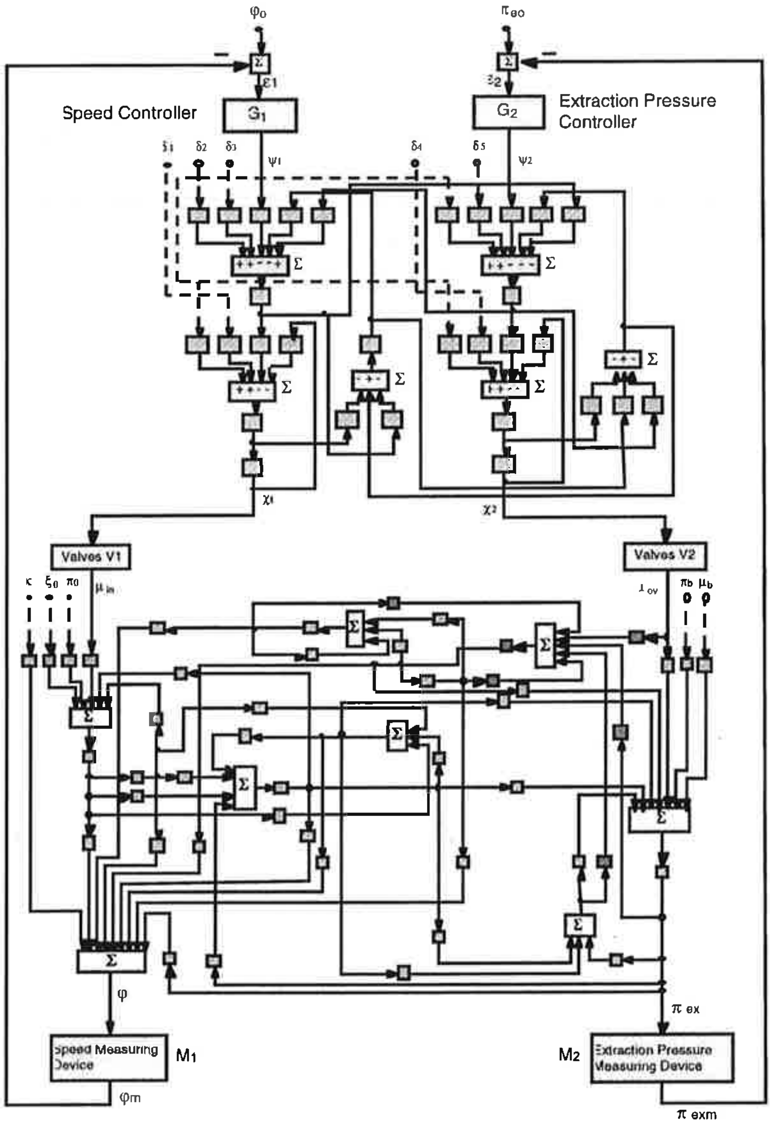

Figure 6: Structural diagram of a power plant´s control system. Two variables have to be controlled. (The symbols of the transfer functions in the shaded blocks are omitted. All non-linearities are linearized.)

2. The Aspect of a Generalized Structure

By minutely investigating the structure of technical, multiple control systems of higher complexity, it becomes apparent that in any particular area within the system an arbitrary number of channels – each channel carrying its individual information – are led into one operational block, where this information undergoes functional treatment. The operational block has only one outlet of information through one channel. This channel branches into an arbitrary number of leads, transporting this information in full to other blocks. This already mentioned situation is illustrated with Figure 6. It represents, as said, a technical power plant with its control system. The two controlled variables, the speed φ (or frequency) of a generator and a steam pressure π_{ex} are the variables that go to their controllers. The pressure π_{ex} is the pressure of extracted steam taken off from a conduit between two turbines, which are driving the generator.

The repeated structural element in Figure 6 is as said: Two to several variables are summed up into Σ-points. The summing point has only one output variable. After passing through a transfer function, this output variable is lead to several transfer functions of which the output goes to other summing points Σ. Figure 7 illustrates this principle. We call Figure 7 an operational block and give it the symbolic name (ΣK).

In Figure 7, the signals x_i enter the operational block, (ΣK), after they run through their individual transfer functions F_i. Then they are summed up (super-imposed) as ΣK. The total information undergoes a further transformation by the transfer function G, before the result will branch into a series of channels. All these channels are carrying the same output variable y_i. Every channel y_i leads its variable to a next transfer function, which is part of the next operational block (ΣK).

It is of our interest to determine the number of closed loops, which can be traced within an arrangement of a larger system, and to take this number as a measure of the complexity of that system. Therefore, as mentioned, Figure 4 has a complexity of 12.

The brain structure of mammals, as shown in Figure 1 (simplified), has about the same structure as shown in Figure 7, but in which the transfer function G and the Σ–point are packed into a neuron, as indicated in Figure 7.

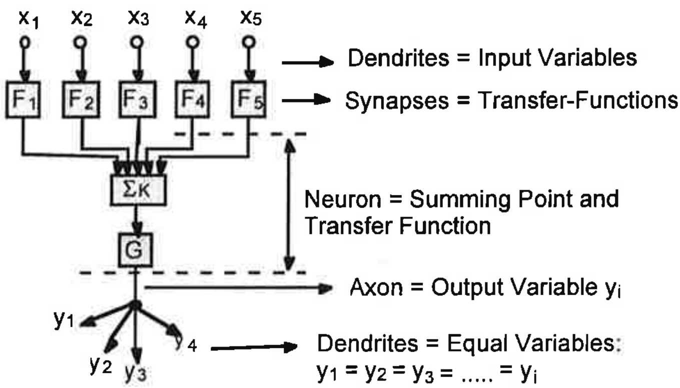

Figure 7: The operational block (ΣK) in Figure 6.

For deriving a formula to calculate the number of closed loops within a structure, a structural generalization is necessary. Referring to Figure 7, this generalization is based on the following procedure: Every output y_i leads (via a transfer function) to all other summing blocks (ΣK). As an example, a system with three operational blocks (ΣK), $(\Sigma 1G_1)$–$(\Sigma 2G_2)$–$(\Sigma 3G_3)$, looks like Figure 8. It renders five individual loops, namely 3 loops containing 2 operational blocks (the operational blocks now without their brackets);

F_{12}-Σ1-G_1-F_{21}-Σ2-G_2,
F_{13}-Σ1-G_1-F_{31}-Σ3-G_3,
F_{23}-Σ2-G_2-F_{32}-Σ3-G_3;

and two loops containing three operational blocks:

F_{12}-Σ1-G_1-F_{31}-Σ3-G_3-F_{23}-Σ2-G_2,
F_{13}-Σ1-G_1-F_{21}-Σ2-G_2-F_{32}-Σ3-G_3.

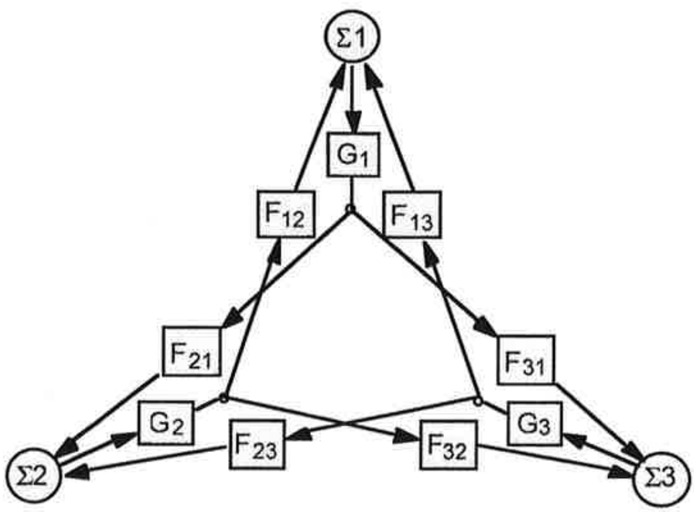

Figure 8: „Brain" of three units Figure 7.

3. The Number of Loops in Growing Systems

a) The 2-(Σ**K**)-system.

The system with 2 (ΣK)s (2 neurons) is basically the structure of the general automatic control system. It is a loop with a continuous circular information flow. Hypothetically, we assume that such a loop is in the brain the elementary quantum of memory. By opening a loop, or when part of it dies, its memory gets lost. We also assume that each neuron is a sub-brain that has its own internal complexity.

b) The 3-(Σ**K**)-system.

With forming a triangle with three operational blocks, Figure 8 evolves. There are 3 loops containing 2 **Σ**–points and 2 loops, containing 3 **Σ**–points, as already mentioned above.

c) The 4-(ΣK)-system.

In the system with four blocks (**Σ**K), already 20 loops can be counted, loops which have 2, 3, and 4 **Σ**–points. Schematically, the system looks like Figure 9. For more clarity, the 20 loops are depicted with Figure 10.

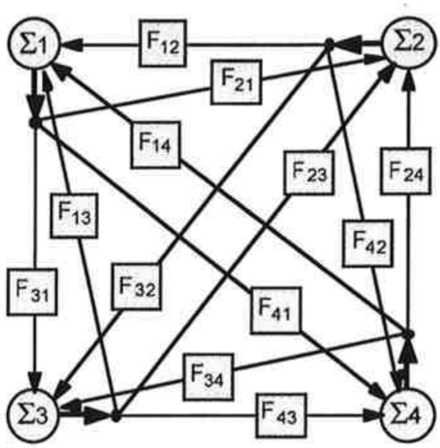

Figure 9: The 4-(ΣK)-System.

[c1] The loops which contain 2 Σ–points can be found with the formula n(n–1)/2, where n is the number of the Σ–points (ΣK) of the system. With n = 4, the number of loops, therefore, is 6.

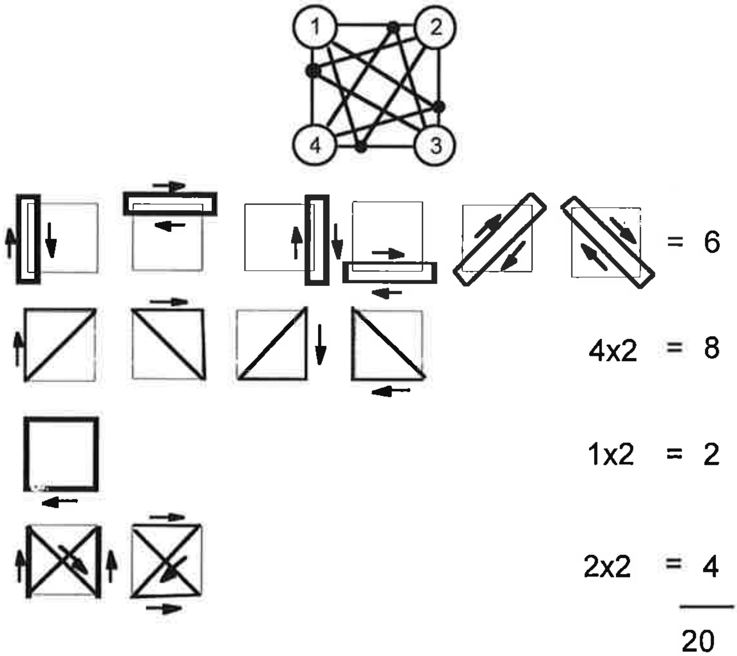

Figure 10: All possible loops in a 4-(ΣK)-System.

[c2] Loops containing 3 Σ–points run through two sides of the quadrilateral. If this triangle rotates around the quadrilateral, 4 additional loops result. But because loops with more than 2 Σ–points show a clockwise and a counter–clockwise flow of information, the result is $2n|_{n=4} = 8$ loops.

[c3] Loops of 4 Σ–points size the whole figure already. Due to the two senses of information flow, 2 loops result.

[c4] But loops of 4 Σ–points can be twisted and as there, again, are two senses of flow, 4 more loops result. Thus, the sum of loops for Figure 10 is **20**.

d) The 5-(ΣK)-system.

This system renders 84 possible loops, which can be found from Figure 11.

e) The 6-(ΣK)-system.

The variety of loop forms is considerably increased in this case. The total number of possible loops was found to be 409.

4. The Derivation of the Formula:
The n-(ΣK)-System

In pursuing systematically the generation of the number of loops, the following pattern develops:

The 2-(ΣK)-system, that is the combination of 2 (ΣK)s, renders 1 loop.
Say: 1 combination of 2 (ΣK)s equals 1! loop; „1 factorial loops".

The system of 3 (ΣK)s is built with 3 combinations of 2 (ΣK)s of 1 loop, plus 1 combination of 3 (ΣK)s of 2 loops.
Say: 3 combinations of 2 (ΣK)s equals 3*1! loops = 3 loops, plus 1 combination of 3 (ΣK)s equals 1*2! loops = 2 loops.
The sum is 5 loops.

The system of 4 (ΣK)s is built with 6 combinations of 2 (ΣK)s. This equals 6*1! = 6 loops. In addition, there are 4 combinations of 3 (ΣK)s, equals 4*2! = 8 loops. Finally, there is 1 combination of 4 (ΣK)s, equals 1*3! = 6 loops.
The sum is 20 loops.

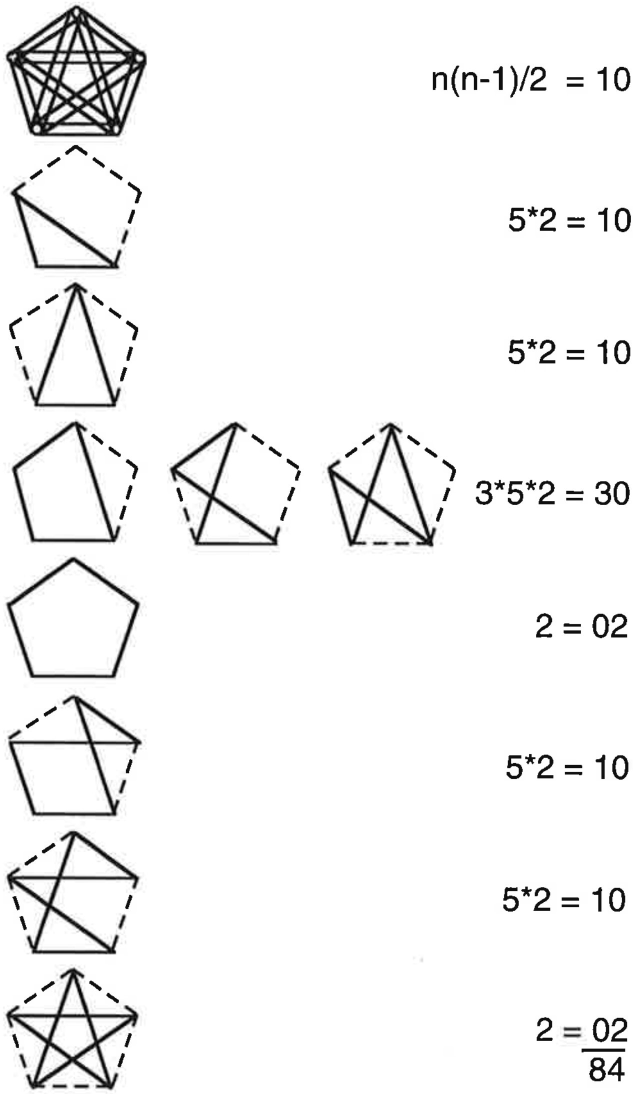

Figure 11: All loops in a 5-(ΣK)-System.

The system of 5 (ΣK)s is built with 10 combinations of 2 (ΣK)s = 10* 1! = 10 loops;
plus 10 combinations of 3 (ΣK)s = 10*2! = 20 loops;
plus 5 combinations of 4 (ΣK)s = 5*3! = 30 loops;
plus 1 combination of 5 (ΣK)s = 1*4! = 24 loops.
The sum is 84 loops.

The system of 6 (ΣK)s. The routine is now:
15 combinations of 2 (ΣK)s = 15*1! = 15 loops;
20 combinations of 3 (ΣK)s = 20*2! = 40 loops;
15 combinations of 4 (ΣK)s = 15*3! = 90 loops;
6 combinations of 5 (ΣK)s = 6*4! = 144 loops;
1 combination of 6 (ΣK)s = 1*5! = 120 loops.
The sum is 409 loops.

The system of 7 (ΣK)s:
21 combinations of 2 (ΣK)s = 21*1! = 21 loops;
35 combinations of 3 (ΣK)s = 35*2! = 70 loops;
35 combinations of 4 (ΣK)s = 35*3! = 210 loops;
21 combinations of 5 (ΣK)s = 21*4! = 504 loops;
7 combinations of 6 (ΣK)s = 7*5! = 840 loops;
1 combinations of 7 (ΣK)s = 1*6! = 720 loops.
The sum is 2´365 loops.

From the figures that give the number of the individual combinations, it is easy to recognize the binomial coefficients of Pascal's triangle, Figure 12. The number of the basic combination can be found by the combination of n (ΣK)s = (n–1)! loops.

From the coefficients of Pascal's triangle, the two last ones are not used. Defining 0! and (–1)! as zero, the total lines of binomial coefficients can be used. The total number of possible loops A_n of a generalized interconnected net of n (ΣK)s can, therefore, be determined by

$$A_n = \sum_{i=1}^{n-1} a_i (n-i)!; \; n \geq 2$$

where a_i is the i-th binomial coefficient of the n-th line in Pascal's triangle. The coefficient a_i can be determined with the formula

$$a_i = \binom{n}{i-1}.$$

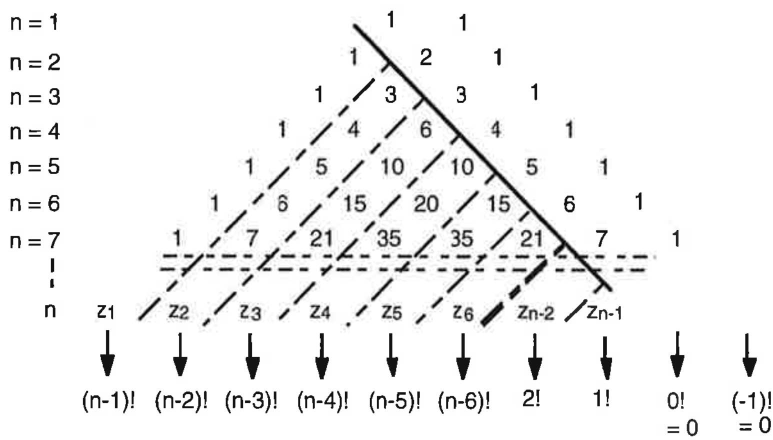

Figure 12: Pascal's triangle for determining the coefficients.

Thus:

$$A_n = \sum_{i=1}^{n-1} \binom{n}{i-1} (n-i)! \quad n \geq 2. \tag{1}$$

Table I gives the number of loops A_n for systems with n (ΣK)s (neurons) from n = 2 till n = 20 neurons: $2 \leq n \leq 20$.

Table I: Number of loops A_n for systems with n (ΣK)s from n = 2 till n = 20: $2 \leq n \leq 20$.

n	A_n
2	1
3	5
4	20
5	84
6	409
7	2´365
8	16´064
9	125´664
10	1´112´073
11	10´976´173
12	119´481´284
13	1´421´542´628
14	18´348´340´113
15	255´323´504´917
16	3´809´950´976´992
17	60´683´990´530´208
18	1´027´542´662´934´897
19	18´430´999´766´219´317
20	349´096´684´728´623´316

The form of the formula (1) is not convenient. In Appendix A it is transformed into

$$A_n = n!\sum_{k=0}^{n-2}\frac{1}{k!(n-k)};\ n \geq 2;\ (0! = 1) \qquad (2)$$

As an example for a system of 12 neurons, we find A_n with the formula (2):

$$A_{12} = 12!\left[\begin{array}{c}\frac{1}{0!12}+\frac{1}{1!11}+\frac{1}{2!10}+\frac{1}{3!9}+\frac{1}{4!8}+\frac{1}{5!7}+\\ \frac{1}{6!6}+\frac{1}{7!5}+\frac{1}{8!4}+\frac{1}{9!3}+\frac{1}{10!2}\end{array}\right]=119'481'284$$

The following question might be of interest. What is the growth of ΔA_{n+1} from A_n to A_{n+1}?

$$\Delta A_{n+1} = A_{n+1} - A_n \qquad (3)$$

The answer, which is developed in Appendix B, is

$$\Delta A_{n+1} = n!\sum_{k=0}^{n-1}\frac{1}{k!}. \qquad (4)$$

As an example for ΔA_7:

$$\Delta A_7 = 6!\sum_{k=0}^{n-1}\frac{1}{k!} = 720\left[\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}\right] = 1956.$$

Table I indicates that the difference between the two loop numbers, $n_7 = 2365$ and $n_6 = 409$, is indeed 1956.

In another way, the question can be asked: Having a tiny little brain of 30 neurons and interconnecting one more neuron to them to form a system of 31 neurons, how many more loops would be formed and could be tracked?

Instead of developing the equation (4), we can take a short cut. We know that

$$e = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\ldots+\frac{1}{p!};\ p=\infty.$$

(e is the basis of the natural logarithm.)

This series is converging rapidly. Therefore, the series can be set equal to $\sum_{k=0}^{n-1}\frac{1}{k!}$ and n = 30; $\frac{1}{0!}=1$.

The equation (4), that is $\Delta A_{n+1} = n!\sum_{k=0}^{n-1}\frac{1}{k!}$, becomes

$$\Delta A_{31} = 30!e.$$

Adding one more neuron to a system of 30 neurons, about 265'252859'812191'058636'308480'000000.e ≈ $7.2*10^{32}$ additional loops will be created! This is extremely surprising! Adding one neuron to a brain of 30 neurons creates $7.2*10^{32}$ additional loops.

It also can be shown – as this is done in Appendix C – that

$$A_{n+1} = (n+1)A_n - nA_{n-1} + n \qquad (5)$$

with the definitions $A_0 = 0$ and $A_1 = 0$.

Examples, starting at the very bottom, i.e., with A_2:

$$\begin{aligned}
A_2 &= 2A_1 - 1A_0 + 1 = 1\\
A_3 &= 3A_2 - 2A_1 + 2 = 5\\
A_4 &= 4A_3 - 3A_2 + 3 = 20\\
A_5 &= 5A_4 - 4A_3 + 4 = 84\\
A_6 &= 6A_5 - 5A_4 + 5 = 409\\
A_7 &= 7A_6 - 6A_5 + 6 = 2'365
\end{aligned}$$

With this sequence of formula (5), it is possible to find the accurate number of loops for a brain of 30 neurons without needing a computer that can handle numbers of the size 10^{36}.

In addition, it can be shown that

$$A_{n+1} = A_n n; \text{ or } \frac{A_{n+1}}{A_n} = n_{n\to\infty}, \qquad (6)$$

or

$$A_n \sim nA_{n-1};\ n \to \infty,$$

or

$$A_n \sim n!;\ n \to \infty. \qquad (7)$$

Formula (6) is derived in Appendix D. It says that the growth of complexity with increasing numbers of neurons is roughly n!.

5. A Point of View

The essay demonstrates that a brain of 30 neurons, which are generally interconnected with each other, leads to such a number of one–loop systems that, if counting for each loop 1 mm^2 and forming a carpet, this carpet would cover the surface of our globe 50´000´000´000 times. And as a comparison, an ant already has a brain of about 60´000 neurons!

The human brain, indeed, does not have a mere 60 000 neurons, but approximately 10^{15} (The exponent 15 is uncertain). Without assuming a generalized interconnectedness – which, indeed, would mean complete chaos – but allowing different "transfer functions" leading to and coming from a neuron, and specific interrelations among neurons in different individuals, a still inconceivable number of different beings can occur, thus resulting in different behaviour. From such a standpoint, it can be said that there will never be two equal human beings on earth (not even after cloning), and that the potential for differentiation is far beyond our perception.

Figure 13 depicts a generalized interconnectedness of 12 neurons. It illustrates the dense web of 119´481´284 loops. Each neuron has 11 inputs and one output. As neurons in the human brain can have thousands of inputs via dendrites, it can or must be assumed that each neuron in our brain is, as mentioned, already a tiny brain, a sub-brain in itself. The application of the formula (2) leads to the number 119´481´284. This was already shown above.

The number of loops A_{30} for a brain of 30 neurons, generated with the formula (2) and n = 30,

$$A_n = n!\sum_{k=0}^{n-2}\frac{1}{k!(n-k)}; \; n \geq 2; \; (0! = 1) \qquad (2)$$

would have about 32 digits, too many for an old fashioned home computer! However, the picture Figure 14 illustrates the dens web in which so many loops could be traced that 1 mm^2 per loop would cover the earth $5*10^{10}$ times.

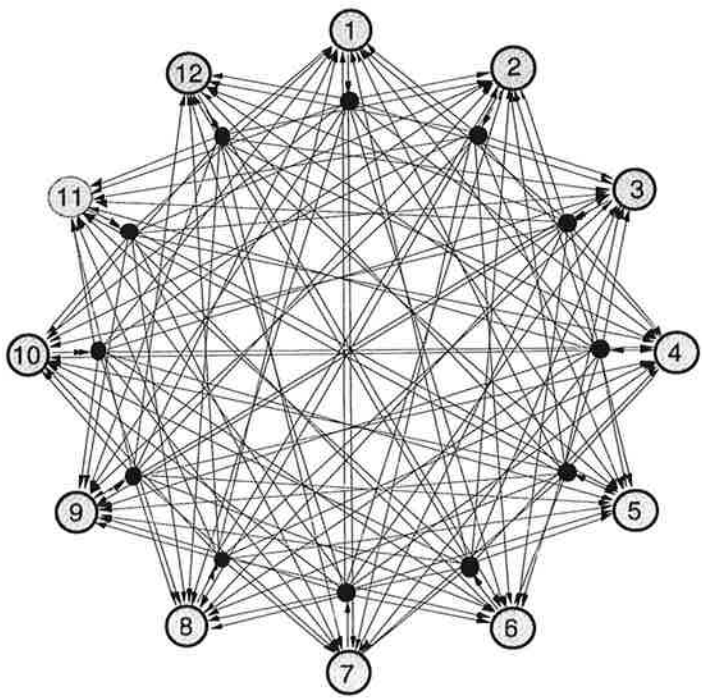

Figure 13: Structure of 12 neurons in generalized interconnectedness.

The number of loops A_{12} is 119′481′284.

We see here an important law of nature: With relatively few elements, she creates an enormous variety of combinations. A similar example can be seen in the construction of languages: With 30 letters of the alphabet, 100 times 100´000 different words can be formed; and with each package of 100´000 words a specific language can be created. Each language establishes an inconceivable amount of ideas and illusions.

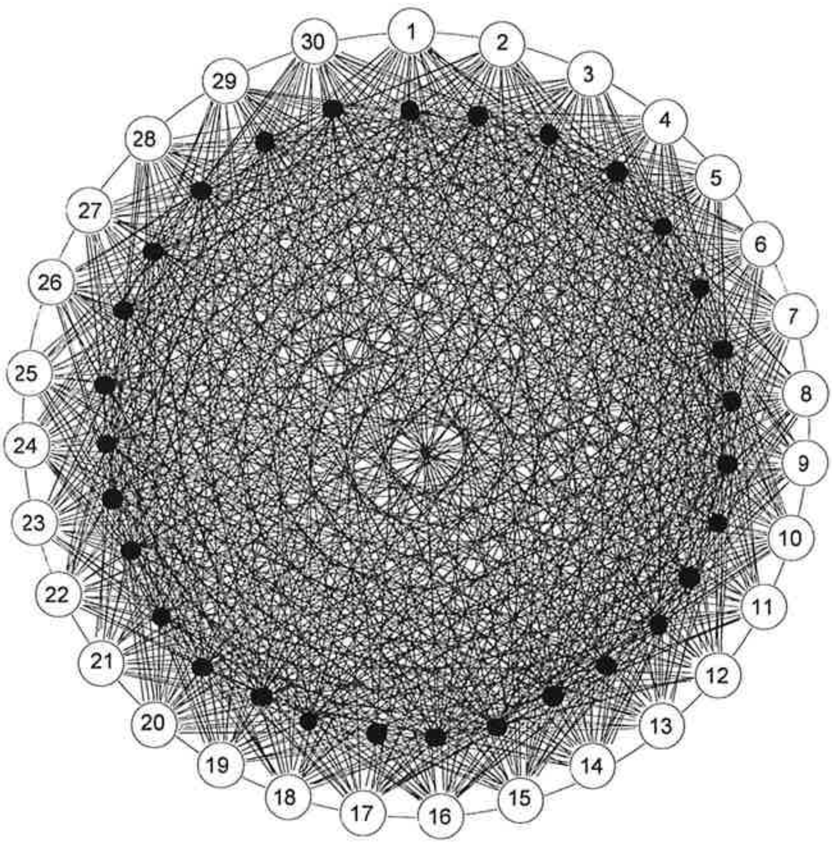

Figure 14: 30 generally interconnected neurons.
The number of loops is roughly
720´000000´000000´000000´000000´000000.

Further well-known examples are the immense world of music, composed with the spectra of air vibrations of a few scales, and the enormous combinations of the DNA-structure.

6. The Dynamics

The structural formation offers a basic insight into the complexity of a system. However, a structure as such is merely a rigid picture. It does not manifest any form of life. It is rather the time-dependent motion, that is the dynamics, that represents living behaviour. Therefore, a short view into the dynamics of a simple structure shall demonstrate that the pure architectural structure has only a limited value. The dynamic view gives a perspective to what extent a complexity increases from the inflexible, structural form to the dynamic-living behaviour.

Assuming that Figure 15 represents a dualism in an utmost rudimentary form. L_1 and L_2 shall represent two persons. Each person is depicted as a goal striving, automatic control loop. L_1´s goal is u_1, L_2´s goal is u_2. The goal is the person´s ultimate endeavour to exist. G_1 and G_2 are the persons' will to achieve the goal. S_{11} and S_{22} represent the person's unconscious part that is related to the goal-striving process. F_1 and F_2 comprise the time-dependent dynamics that make L_1 and L_2 capable to constantly operate. R_1 and R_2 are the signals providing self-reflection, which is the awareness to exist. The feeling of existing comes from moving in time. This self-reflective action from the unconscious to the consciousness via the feedback signals (R_1 and R_2) in the time of living is providing the „I think, therefore I am“ (the „cogito ergo sum“). The two disturbance signals, d_1 and d_2, in Figure 15 shall be disregarded for our purpose.

The triple bilateral connections of the two persons stand for the following information exchange between them. In short:

S_{12} and S_{21} are the bilateral, unconscious attitude transfer functions. They exchange the attitude the two persons harbour for or against each other. The attitude variables, $S_{12}\varphi_2$ and $S_{21}\varphi_1$, make the dualism either friendly or antagonistic, depending whether the product $S_{12}S_{21}$ is negative or positive; [1].

The $V_{12}x_2$- and $V_{21}x_1$-variables stand for the mutual observation of each other. Observation can be based on forbearance or spying; [2].
The $A_{12}\delta_2$- and $A_{21}\delta_1$-variables represent physical actions, be they discussing, arguing or fighting; [3, Appendix III].

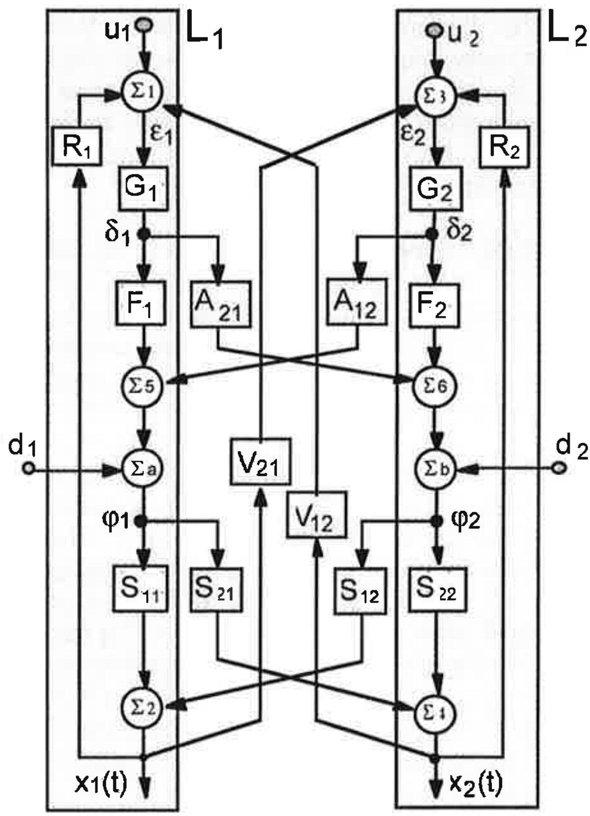

Figure 15: Principle structure of a dynamic liaison of two persons, L_1 and L_2.

The three bilateral information exchanges, which connect the two loops, are time-dependent. Similarly time-dependent are F_1 and F_2 and R_1 and R_2, because nothing can happen without some time. Time -dependency is described with differential equations. But we leave such mathematical material packed in symbols for all the 14 transfer functions in Figure 15. These functions are G_1 and G_2, F_1 and F_2, S_{11}, S_{22}, S_{12}, and S_{21}, R_1 and R_2, V_{12} and V_{21}, and A_{12} and A_{21}.

Referring to our view of structural formations, every block (ΣK) receives two or three input signals, but has only one output.

If a system is living, it performs this action with a demonstration of behaviour. The behaviour is a function of the system´s character. The character is a function of the system´s constituents. In our example of Figure 15, these constituents are the 14 characteristic transfer functions G_1, G_2, F_1, F_2, S_{11}, S_{22}, R_1, R_2, S_{12}, S_{21}, V_{12}, V_{21}, A_{12}, and A_{21}. Assuming further that all time-dependent characteristics are mathematically known in the form of linearized differential equations, the character of the dualism Figure 15, that is its characteristic equation, can be determined. Then the dualism´s behaviour is known in all its details. (The linearization is necessary due to the addition of the variables in the (ΣK)s).

An insight into this characteristic equation gives an idea how turbulent it can become when two people get together forming a liaison. The characteristic equation of Figure 15 is depicted with equation (8). Indeed as mentioned, each one of the 14 symbols in the equation is replacing a differential equation of some order.

The two small parts in bold set in the equation mean to be the autonomous character of the persons. These are the equations (9). The first equation is for L_1, the second for L_2.

$$\begin{aligned} G_1F_1S_{11}R_1 + 1 &= 0 \\ G_2F_2S_{22}R_2 + 1 &= 0 \end{aligned} \tag{9}$$

(It is noteworthy that the description of a person's character fills a small booklet and is not done with merely four symbols!)

$$
\begin{aligned}
1 &- \mathbf{G_1F_1S_{11}R_1} - G_1A_{21}S_{12}R_1 - G_2A_{12}S_{11}V_{21} - G_2F_2S_{12}V_{21} \\
&- G_1F_1S_{21}V_{12} - G_1A_{21}S_{22}V_{12} - G_2A_{12}S_{21}R_2 - \mathbf{G_2F_2S_{22}R_2} \\
&+ (G_2F_2S_{12}V_{21})(G_1F_1S_{21}V_{12}) \\
&+ (G_2A_{12}S_{11}V_{21})(G_1A_{21}S_{22}V_{12}) \\
&+ (G_1A_{21}S_{12}R_1)(G_2A_{12}S_{21}R_2) \\
&+ \mathbf{(G_1F_1S_{11}R_1)(G_2F_2S_{22}R_2)} \\
&- G_1G_2F_1F_2S_{12}S_{21}R_1R_2 \\
&- G_1G_2A_{12}A_{21}S_{11}S_{22}R_1R_2 \\
&- G_1G_2A_{12}A_{21}S_{12}S_{21}V_{12}V_{21} \\
&- G_1G_2F_1F_2S_{11}S_{22}V_{12}V_{21} = 0 \qquad (8)
\end{aligned}
$$

All the other parts in the equation (8) are due to the formation of the liaison. Firstly, we see that the two characters (9) become added; secondly, they become multiplied with each other -- shown also in bold setting. In addition, it can be seen how tremendously interrelated the dualism must behave with its three bilateral exchanges of information; how they interact with the two persons and within them. We also point to the fact that in the last eight lines of the equation (8) the two wills of the two persons, G_1 and G_2, are multiplied with each other. This product, G_1G_2, is a highly explosive package for maintaining stability, for keeping the liaison in a homeostatic state. The fact that two characters are added and multiplied with each other cannot be comprehended with our brain, although in the technical world it is a common and realistic fact. It must be the same in living beings. This unfathomability makes a mutual understanding of L_1 and L_2 impossible, because nobody can comprehend a something of which he is only a part. The unperceivable interconnectedness is the eternal seed for conflicts!

Putting the time-behaviour of these 14 characteristic expressions into their explicit, time -dependent form would extend the equation (8) easily to a length of 100 meters. And the number of loops, our loop-complexity, is 12 only, as is illustrated with Figure 5. This bit on the side of the topic makes foreseen how incredibly complex our world is formed - far beyond any human comprehension. And, what makes the situation worse, a brain is to an unknown amount an independent living being in every mammal's head. This brain is in constant change of adapting and creating.

My mother used to say:

Träume sind Schäume!
Dreams are foams!

She was wrong. Your dreams are specific bits of information, explanations, and declarations from our unconscious to your consciousness - when we wake up, remember, and if we understand the dreams meaning.

References from the Author with respect to Chapter 6.

[1] Starkermann, R., Die demokratische Hierarchie. Modell einer sozialen Verheerung", Feedcross-Verlag, Editions à la Carte, Zürich, 2009, ISBN: 978-3-905708-53-0.

[2] Starkermann, R., Amity and Enmity - Two Archetypes of Social Existence, Volume III; Feedcross-Verlag, Editions à la Carte, Zürich, 2004, ISBN: 3-908730-81-3.

[3] Starkermann, R., Amity and Enmity - Two Archetypes of Social Existence, Volume I; Feedcross-Verlag, Editions à la Carte, Zürich, 2003, ISBN: 3-908730-29-5.

Appendix A

We show the derivation of the modus operandi of the essay, the formula (2):

$$A_n = n!\sum_{k=0}^{n-2}\frac{1}{k!(n-k)};\ n \geq 2;\ (0! = 1). \qquad (2)$$

If in the expression

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \qquad (A1)$$

k is changed to (i-1), the expression for the coefficient a_i changes to

$$a_i = \binom{n}{i-1},$$

and to the formula (1):

$$A_n = \sum_{i=1}^{n-1}\binom{n}{i-1}(n-i)!\ \ n \geq 2. \qquad (1)$$

With (A1) set in (1), the expression (A2) is formed:

$$A_n = \sum_{i=1}^{n-1}\frac{n!(n-i)!}{(i-1)!(n-i+1)!}, \qquad (A2)$$

or

$$A_n = n!\sum_{i=1}^{n-1}\frac{(n-i)!}{(i-1)!(n+1-i)!}.$$

The fraction can be cancelled down by (n-i)!. We get (A3):

$$A_n = n!\sum_{i=1}^{n-1}\frac{1}{(i-1)!(n+1-i)}. \qquad (A3)$$

By setting i-1 = k in (A3), we get to the expression (2):

$$A_n = n!\sum_{k=0}^{n-2}\frac{1}{k!(n-k)}; \; n \geq 2; \; (0! \; = \; 1). \qquad (2)$$

An explicit example for A_{12} is given in the text, Chapter 4.

Appendix B

The question to be answered is: What is

ΔA_{n+1} from A_n to A_{n+1}?

By applying formula (2)

$$A_n = n!\sum_{k=0}^{n-2}\frac{1}{k!(n-k)}; \; n \geq 2; \; (0! \; = \; 1) \qquad (2)$$

equation (3)

$$\Delta A_{n+1} = A_{n+1} - A_n \qquad (3)$$

becomes:

$$\Delta A_{n+1} = (n+1)!\sum_{k=0}^{n-1}\frac{1}{k!(n+1-k)} - n!\sum_{k=0}^{n-2}\frac{1}{k!(n-k)}$$

or

$$n!\left[(n+1)\sum_{k=0}^{n-1}\frac{1}{k!(n+1-k)} - \sum_{k=0}^{n-2}\frac{1}{k!(n-k)}\right].$$

This form is equal to

$$\Delta A_{n+1} = n!\left[1 + \sum_{k=1}^{n-1}\frac{n+1}{k!(n+1-k)} - \sum_{k=0}^{n-2}\frac{1}{k!(n-k)}\right]. \qquad (B1)$$

Replacing k with k+1 in the negative term of the bracket, that is in the term

$$\sum_{k=0}^{n-2}\frac{1}{k!(n-k)},$$

and by changing its limits from $\sum\limits_{k=0}^{n-2}$ to $\sum\limits_{k=1}^{n-1}$, the expression (B1) becomes

$$\Delta A_{n+1}=n!\left[1+\sum_{k=1}^{n-1}\frac{(n+1)(k-1)!-k!}{k!(k-1)!(n-k+1)}\right]$$

or

$$\Delta A_{n+1}=n!\left[1+\sum_{k=1}^{n-1}\frac{(n+1)(k-1)!-(k-1)!k}{k!(k-1)!(n-k+1)}\right],$$

or

$$\Delta A_{n+1}=n!\left[1+\sum_{k=1}^{n-1}\frac{(k-1)!(n+1-k)}{k!(k-1)!(n-k+1)}\right].$$

Therefore, we get:

$$\Delta A_{n+1}=n!\left[1+\sum_{k=1}^{n-1}\frac{1}{k!}\right]=n!\sum_{k=0}^{n-1}\frac{1}{k!}.$$

$$\Delta A_{n+1}=n!\sum_{k=0}^{n-1}\frac{1}{k!}. \qquad (4)$$

Appendix C

It is to be demonstrated that

$$A_{n+1} = (n+1)A_n - nA_{n-1} + n. \qquad (5)$$

Starting with the two formulae (3) and (4),

$$\Delta A_{n+1} = A_{n+1} - A_n \tag{3}$$

$$\Delta A_{n+1} = n!\sum_{k=0}^{n-1}\frac{1}{k!}. \tag{4}$$

we get

$$\Delta A_{n+1} = n!\sum_{k=0}^{n-1}\frac{1}{k!} = n! + n!\sum_{k=1}^{n-1}\frac{1}{k!} = n!\left[1+\sum_{k=1}^{n-1}\frac{1}{k!}\right];\ \left[\frac{1}{0!}=1\right].$$

$$\Delta A_{n+1} = (n-1)!\sum_{k=0}^{n-2}\frac{1}{k!};$$

derived from (4). Therefore,

$$\Delta A_{n+1} = (n-1)!n\left[\sum_{k=1}^{n-2}\frac{1}{k!} + \frac{1}{(n-1)!}\right] = (n-1)!\ n\sum_{k=0}^{n-2}\frac{1}{k!} + n;$$

$$\Delta A_{n+1} = n\underbrace{\left[(n-1)!\ n\sum_{k=0}^{n-2}\frac{1}{k!} + 1\right]}_{\Delta A_n + 1}.$$

Thus:

$$\Delta A_{n+1} = n(\Delta A + 1),$$

and

$$A_{n+1} - A_n = n[A_n - A_{n-1} + 1].$$

Thus:

$$A_{n+1} = (n+1)A_n - nA_{n-1} + n; \tag{5}$$

$$\text{Def.: } A_0 = 0,\ A_1 = 0.$$

Appendix D

Proof of equation (6) i.e. $A_n = nA_{n-1}$; for $n \rightarrow \infty$

$$A_n = nA_{n-1} \text{ or } \frac{A_n}{A_{n-1}} = n_{n\rightarrow\infty}.$$

We know that

$$A_n = \sum_{i=1}^{n-1} \binom{n}{i-1} (n-i)! \quad n \geq 2. \tag{1}$$

Therefore,

$$\frac{A_n}{A_{n-1}} = \frac{\sum_{i=1}^{n-1} \binom{n}{i-1} (n-i)!}{\sum_{i=1}^{n-2} \binom{n-1}{i-1} (n-1-i)}.$$

Using the form

$$\binom{n}{k} = \frac{n!}{k!(n-k)!},$$

we get

$$\frac{\sum_{i=1}^{n-1} \frac{n!(n-i)!}{(i-1)!(n-i+1)!}}{\sum_{i=1}^{n-2} \frac{(n-1)!(n-1-i)!}{(i-1)!(n-1-i+1)!}} = \frac{\sum_{i=1}^{n-1} \frac{n!}{(i-1)!(n-i+1)}}{\sum_{i=1}^{n-2} \frac{(n-1)!}{(i-1)!(n-i)}}. \tag{D1}$$

Extending the two Σ-expressions, namely the dividend with n! and the divisor with (n-1)!:

$$\frac{n!\left[\frac{1}{0!n}+\frac{1}{1!(n-1)}+\frac{1}{2!(n-2)}+\frac{1}{3!(n-3)}+\ldots+\frac{1}{(n-3)!3}+\frac{1}{(n-2)!2}\right]}{(n-1)!\left[\frac{1}{0!(n-1)}+\frac{1}{1!(n-2)}+\frac{1}{2!(n-3)}+\frac{1}{3!(n-4)}+\ldots+\frac{1}{(n-4)!3}+\frac{1}{(n-3)!2}\right]}$$

we get:

$$\frac{A_n}{A_{n-1}}=n\frac{\left[\frac{1}{0!}+\frac{1}{1!\left(1-\frac{1}{n}\right)}+\frac{1}{2!\left(1-\frac{2}{n}\right)}+\frac{1}{3!\left(1-\frac{3}{n}\right)}+\ldots.+\frac{1}{(n-3)!\frac{3}{n}}+\frac{1}{(n-2)!\frac{2}{n}}\right]}{\left[\frac{1}{0!\left(1-\frac{1}{n}\right)}+\frac{1}{1!\left(1-\frac{2}{n}\right)}+\frac{1}{2!\left(1-\frac{3}{n}\right)}+\ldots.+\frac{1}{(n-4)!\frac{3}{n}}+\frac{1}{(n-3)!\frac{2}{n}}\right]}$$

In making the transfer $n\rightarrow\infty$,
then for $(n-3)!\frac{3}{n}$, the term $(n-4)!3$ can be set;
for $(n-2)!\frac{2}{n}$, the term $(n-3)!2$ can be set; etc.

Thus:

$$\frac{A_n}{A_{n-1}}_{n\rightarrow\infty}=n\frac{\left[\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots.+\sim 0+\sim 0\right]}{\left[\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots.+\sim 0+\sim 0\right]}=n\frac{e}{e}=n$$

and the result is:

$$\frac{A_n}{A_{n-1}}_{n\rightarrow\infty}=n.$$

For a big number n it follows:

$$A_n \sim nA_{n-1}$$

or

$$A_n \sim n!$$

Closing Words

The perspectives we experience from the stellar universe down to our globe with its variety of life, from there to the world of molecules, and finally to the subatomic realm, they all reveal a dramatic diversity of phenomena. It is impossible for a human being to comprehend such an immense complexity. A three dimensional boundlessness of the infinite space with its countless galaxies is already incomprehensible for our limited brain. Although we are aware of the colossal variety of events on the globe, of the past and of the present, we are still fully outdistanced from knowing the inherent functioning of this immenseness. Already our view of the architectural structure of the human brain and of our animal fellows must make us conscious about our helplessness in conceiving the world´s marvel.

With the automatic control loop of Figure 3, a vague idea of a single loop was presented in regard to the heating system of a room. However, including the thorough dynamic functioning of such a one-variable automatic control loop would require the study-time of an entire university year, loaded with mathematics and physics.

With the continuing progress in biology, especially in neuro-biology with the newest knowledge of plasticity and neuro-genesis, the certainty grows that even what we call feelings, spirit, mind, and faiths are based on physical-biological functioning. But the real discernment of such functioning, especially the time-dependent behaviour involving homeostasis, is still out of reach for us for a very long time to come. We mentioned that we still know nothing yet about the very operations of the single neuron. And our brain consists of billions of billions of those tiny interacting things. We risk assuming that each neuron could be a sub-brain within the large brain, because a single neuron can receive up to thousand different signals of which it is capable to handle. It, then, sends the response, its output, out through its axon into innumerable directions to supply other neurons with its produced information.

For starting life, nature supplies us with enough knowledge to survive within a certain territory and to live in it for a certain period of time. This knowledge is, unfortunately, by far not enough to establish - what we are longing for - peace. This is so, because everything is interrelated with everything, consciously and unconsciously, and nobody is knowable enough to behave and act within this huge, mysterious network without running into conflicts with himself, with his neighbours and with the world at large. Looking into the world beyond the human realm, a peace, according to our imagination, cannot be found anywhere. We only see excessive reproduction and intensive struggle for survival. And considering our human history of peace and wars, there is about the same: love and hate for reproduction, struggle for survival, and the poor fellow feeds the wealthy one.

Since long, we make much effort to find a definition for consciousness and an explanation for time. But sooner or later, we come to the conclusion that our brain is way too small to come to crisp with such phenomena. Nevertheless, hopping by staying engaged in serious endeavour, we will eventually find out how miracles should work.

On the other hand, in regard of such overwhelming intricacies, it is understandable that the person, who is devoid of the necessary knowledge, needs a perceivable explanation of the world´s construction within the limits of his beholding. His faiths in the story of Creation and in life in heaven afterwards serve his purpose quite well.